Memorandum

Linear movement

m	= mass	[kg]
d	= linear displacement	[m]
v	= linear speed	[m/s]
a	= linear acceleration	[m/s²]
r	= radius	[m]
p	= pitch	[m]
η	= transmission efficiency	[–]
F	= force	[N]

Angular movement

J	= inertia	[kgm²]
θ	= angular displacement	[rad]
ω	= angular speed	[rad/s]
α	= angular acceleration	[rad/s²]
r	= radius	[m]
Z	= number of teeth	[–]
i	= reduction ratio	[–]
k_v	= viscous damping constant	[Nm/rad/s = Nms]
η	= transmission efficiency	[–]
M	= torque	[Nm]

Force

F	$= m \cdot a$	[N]

Torque

M	$= J \cdot \alpha$	[Nm]
ΔM	= viscous damping $= k_v \cdot \Delta\omega$	[Nm]

Work - Energy

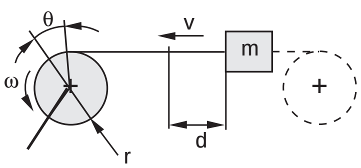
W	$= F \cdot d$	[Nm]
W	$= M \cdot \theta$	[Nm]

Mechanical power

P_m	$= F \cdot v$	[W]
P_m	$= M \cdot \omega$	[W]

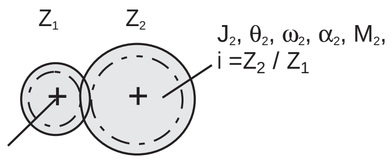
Inertia

Moment of inertia of a ring:	J	$\cong m \cdot r^2$	[kgm²]
Moment of inertia of a cylinder:	J	$= \frac{1}{2} m \cdot r^2 = \pi/2 \cdot r^4 \cdot h \cdot \rho$	[kgm²]
Moment of inertia of a hollow cylinder:	J	$= \frac{1}{2} m (r_1^2 + r_2^2) = \pi/2 \cdot (r_1^4 - r_2^4) \cdot h \cdot \rho$	[kgm²]
	ρ	= specific mass [kg/m³] h = height	[m]

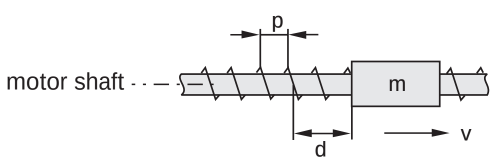


motor shaft

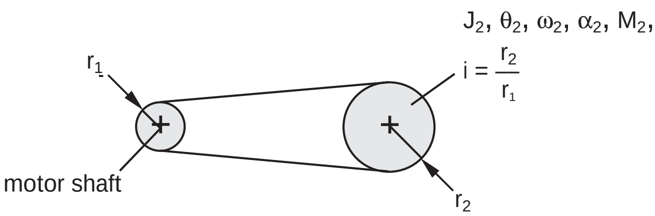
$J = m \cdot r^2$ [kgm²] $M = F \cdot r / \eta$ [Nm]
$\theta = d / r$ [rad]
$\omega = v / r$ [rad/s] $r_{opt.} = \sqrt{J_m/m}$ [m]
$\alpha = a / r$ [rad/s²]



motor shaft



$J = m (p / 2\pi)^2$ [kgm²] $M = F \cdot p / 2\pi \cdot \eta$ [Nm]
$\theta = 2\pi \cdot d / p$ [rad]
$\omega = 2\pi \cdot v / p$ [rad/s] $P_{opt.} = 2\pi\sqrt{J_m/m}$ [m]
$\alpha = 2\pi \cdot a / p$ [rad/s²]



$J_1 = J_2 / i^2$ [kgm²] (load inertia reflected to the motor shaft)
$\theta = \theta_2 \cdot i$ [rad]
$\omega_1 = \omega_2 \cdot i$ [rad/s]
$\alpha_1 = \alpha_2 \cdot i$ [rad/s²]
$M1 = M_2 / i \cdot \eta$ [Nm]
$i_{opt.} = \sqrt{J_2/J_m}$ [–]

API Portescap

Load Data Form

To optimize motor choice and estimate life expectancy for your application, please complete and return a photocopy of the following load data form.

Date:

Company:

Contact person:

Address:

Tel.: Fax:

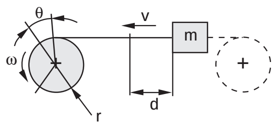
Application / Function: New ☐ Existant ☐

Recommended product:

Load specification

Transmission

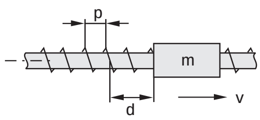
☐ Motor shaft



r = ____ mm

h = ____

☐ Motor shaft

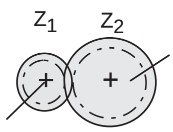


p = ____ mm

h = ____

J = ____ kgm^2

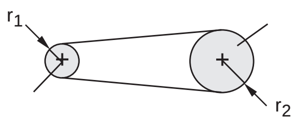
☐ Motor shaft



$i = \frac{Z_2}{Z_1}$ = ____

h = ____

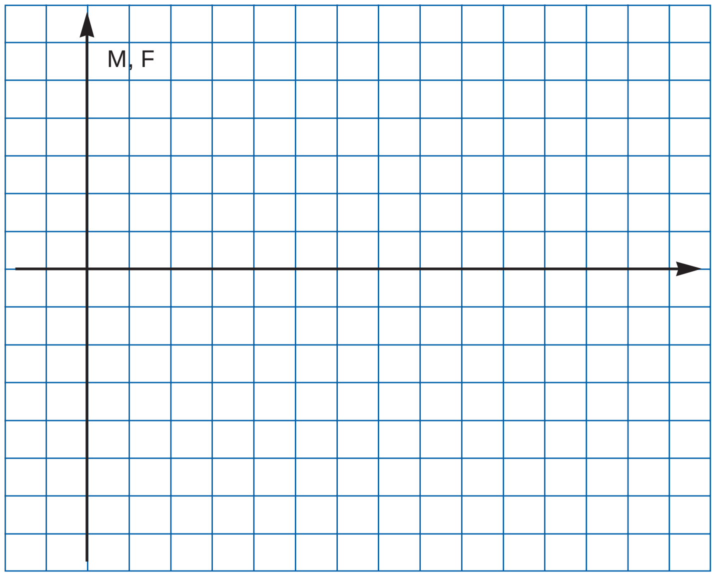
☐ Motor shaft



$i = \frac{r_2}{r_1}$ = ____

h = ____

☐ Other



Mass = ____ kg

Inertia = ____ kgm^2

Static friction force = ____ N

Static friction torque = ____ Nm

Load viscous force = ____ N/m

Viscous damping torque = ____ Nms

Movement specification

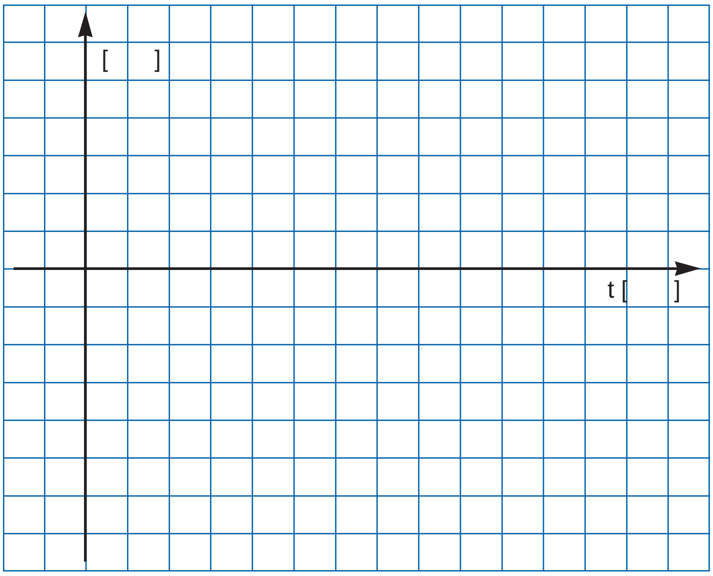
Controlled parameters

Torque: ☐

Speed: ☐

Position: ☐

Trajectory: ☐



Additional constraints:

Precision = ____ [ ]

Resolution = ____ [ ]

Overshoot = ____ %

Application environment

Available power supply: I_{max}= ____ A

Voltage range = ____ V

Available control electronics: ____ f max = ____ Hz

Temperature range: operational = ____ °C storage = ____ °C

Available space: $Ø_{max}$ = ____ mm L_{max} = ____ mm

Lifetime: Operating hours = ____ Number of cycles = ____

Shaft load: axial = ____ N radial = ____ N Distance = ____ mm

Is motor stalled? if yes, conditions: Voltage = ____ V Current = ____ A

Symbols and S.I. units

Conversion table

Symbol	Description	Unit
a	linear acceleration	m/s^2
d	linear displacement	m
f	frequency	Hz
k	torque constant	Nm/A
k_m	motor constant	Nm/$\sqrt{W}$
m	mass	kg
n	rotational speed	rpm
t	time	s
v	linear speed	m/s
B	magnetic induction	T
E	electromotive force	V
F	force	N
H	magnetic field	A/m
I	current	A
J	moment of inertia	kgm^2

Symbol	Description	Unit
L	inductance	H
M	torque	Nm
P	power	W
R	resistance	Ω
R_{th}	thermal resistance	°C/W
T	temperature	°C
U	voltage	V
W	work, energy	Nm
α	angular acceleration	rad/s^2
η	efficiency	–
θ	angular displacement	rad
τ	time constant	s
Φ	magnetic flux	Wb
ω	angular speed	rad/s

Length:	**1 in**	= 25.4	mm	**1 mm**	= 0.0393	in
	1 ft	= 0.3048	m	**1 m**	= 3.281	ft
Mass:	**1 oz**	= 0.0283	kg	**1 kg**	= 35.3	oz
	1 lb	= 0.454	kg	**1 kg**	= 2.205	lb
Force:	**1 kp**	= 9.81	N	**1 N**	= 0.102	kp
	1 oz	= 0.278	N	**1 N**	= 3.597	oz
	1 lb	= 4.45	N	**1 N**	= 0.225	lb
Temperature:	**T [°F]**	= 9/5 $T_{°C}$ +32		**T [°C]**	= 5/9 ($T_{°F}$-32)	
	0 K	= -273.15	°C	**0 °C**	= 273.15	K
Torque:	**1 kpcm**	= 0.0981	Nm	**1 Nm**	= 10.2	kpcm
	1 oz-in	= 7.06	mNm	**1 mNm**	= 0.1416	oz-in
	1 lb-in	= 0.113	Nm	**1 Nm**	= 8.849	lb-in
	1 lb-ft	= 1.356	Nm	**1 Nm**	= 0.7376	lb-ft
Inertia:	**1 gcm^2**	= 1 x 10^{-7}	kgm^2	**1 kgm^2**	= 1 x 10^7	gcm^2
	1 oz-in^2	= 1.83 x 10^{-5}	kgm^2	**1 kgm^2**	= 5.46 x 10^4	oz-in^2
	1 oz-in s^2	= 0.00706	kgm^2	**1 kgm^2**	= 141.6	oz-in s^2
	1 moiss	= 7.06 x 10^{-6}	kgm^2	**1 kgm^2**	= 141643	moiss
	1 lb-in^2	= 0.000293	kgm^2	**1 kgm^2**	= 3418	lb-in^2
	1 lb-in s^2	= 0.113	kgm^2	**1 kgm^2**	= 8.85	lb-in s^2
Energy:	**1 kcal**	= 4187	J	**1 J**	= 0.239	cal
	1 Btu	= 1055	J	**1 J**	= 9.48 x 10^{-4}	Btu
Power:	**1 CV**	= 735	W	**1 kW**	= 1.36	CV
	1 HP	= 746	W	**1 kW**	= 1.34	HP

Examples of motor calculations

DIRECT DRIVE WITHOUT A GEARBOX

A load having a friction torque M of 6 mNm should be driven at a speed of 2000 rpm. The ambient temperature T_{amb} is 30°C. The voltage available is 10 V.

The escap® motor table shows the type 22N to be the smallest motor capable of delivering a torque of 6 mNm continuously. Let's take the model 22N 28-213E.201, which has a measuring voltage of 9V. The characteristics we are mostly interested in are the torque constant k of 12.5 mNm/A and the resistance at 22°C of 10.3 Ω. Neglecting the no-load current, for a torque of 6 mNm the motor current is:

$$I = \frac{M}{k} \qquad [A]\ (1)$$

$$I = \frac{6}{12.5} = 0.48\ A$$

We can now calculate the drive voltage required by the motor, at 22°C, for running at 2000 rpm with a load torque of 6 mNm:

$$U = R \cdot I + k \cdot \omega \qquad [V]\ (2)$$

$$\omega = 2\pi \cdot \frac{n}{60} \qquad [rad/s]\ (3)$$

$$U = 10.3 \cdot 0.48 + 12.5 \cdot 10^{-3} \cdot 209.4 = 7.56\ V$$

We note that the current of 0.48 A is quite close to the rated continuous current of 0.62 A. We should therefore calculate the final rotor temperature (T_r) to make sure it stays below the rated value of 100°C and the voltage required is within the 10 V available. In the formulas, P_{diss} is the dissipated power, R_{Tr} is the rotor resistance at the final temperature and α is the thermal coefficient of the copper wire resistance:

$$\Delta T = T_r - T_{amb} = P_{diss} \cdot R_{th} \qquad [°C]\ (4)$$

$$P_{diss} = R_{Tr} \cdot I^2 \qquad [W]\ (5)$$

$$R_{Tr} = R_{22} \cdot (1 + \alpha\,(T_r - 22)) \qquad [\Omega]\ (6)$$

$$\alpha = 0.0039 \qquad [1/°C]\ (7)$$

$$R_{th} = R_{th1} + R_{th2} \qquad [°C/W]\ (8)$$

The catalogue values for the thermal resistance rotor-body and body-ambient are 5°C/W and 20°C/W, respectively. They are indications for unfavourable conditions. Under «normal» operating conditions (motor mounted to a metal surface, with air circulating around it) we may take half the value for R_{th2}.

By solving equations (4) (5) and (6), we obtain the final rotor temperature T_r:

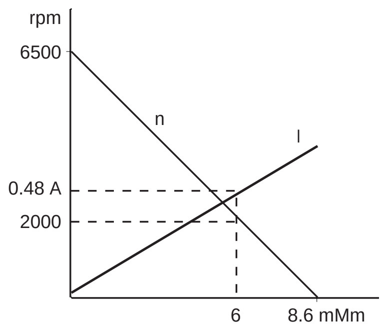
$$T_r = \frac{R_{22} \cdot I^2 \cdot R_{th} \cdot (1-22\,\alpha) + T_a}{1 - \alpha \cdot R_{22} \cdot I^2 \cdot R_{th}} \qquad (9)$$

With a current of 0.48 A the rotor reaches a temperature of:

$$T_r = 72.6°C$$

At that temperature and according to equation (6), the rotor resistance is R_{72} = 12.33 Ω, and we need a drive voltage of 8.5 V. The motor requires an electrical power of 4.1 W.

The problem is now solved. In case the application requires a particularly long motor life, use of the next larger motor (type 22V) could possibly also be considered.



Speed/torque and current/torque lines of the 22N28-213E motor at 68.4°C and for 8.5 V.

The behaviour and basic equations of ironless rotor D.C. motors is described in detail in the technical publication Think escap® 1.

DRIVE USING A GEARBOX

A load with a friction torque of 0.5 Nm should be driven at a speed of 30 rpm.

The gearbox table shows this torque is within the rating of the R22 gearbox. When choosing the reduction ratio we keep in mind that the input speed of the R22 should remain below 5000 rpm in order to assure low wear and low noise emission:

$$i \leq \frac{n_{max}}{n_{ch}} \qquad [-]\ (10)$$

$$i \leq \frac{5000}{30} = 166.7$$

The catalogue indicates a closest ratio of 111:1, the efficiency being 0.6 (or 60%). We may now calculate the motor speed and torque:

$$M_m = \frac{M_{ch}}{i \cdot \eta} \qquad [Nm]\ (11)$$

$$M_m = \frac{0.5}{111 \cdot 0.6} = 7.5 \cdot 10^{-3}\ Nm = 7.5\ mNm$$

$$n_m = n_{ch} \cdot i \qquad [rpm]\ (12)$$

$$n_m = 30 \cdot 111 = 3330\ rpm$$

The motor table shows the 22V motor can deliver 7.5 mNm permanently. The 22V is available as a standard combination with this gearbox. After choosing a winding we calculate the motor current and voltage the same way as in the preceding example.

A very simple graphic procedure of selecting a motor-gearbox unit is presented in the technical publication Think escap® 6.

DRIVE WITH A D.C. MOTOR USING ELECTRONIC COMMUTATION

A torque of 3 mNm is required at a speed of 10 000 rpm, with a life time beyond 15 000 hours. Quite obviously, the best choice is a motor using electronic commutation.

The speed/torque curves show the 26BC-6A-113.101 motor to be able of doing the job. It has an integrated drive circuit, consuming 18 mA which are included in the no-load current. Now let's calculate the necessary current and voltage. The relevant catalogue values are:

equivalent impedance: 6.8 W
torque constant: 9.2 mNm/A
no-load current at 13400 rpm: 110 mA
viscous torque constant: 0.4 • 10-6 Nms

The «equivalent impedance» is the impedance at any two of the three winding terminals. It cannot be measured from outside because of the presence of the driver transistors.

The change in friction caused by a speed change is given by the viscous damping constant k_v:

$$k_v = \frac{\Delta M_f}{\Delta \omega} \qquad [Nm/rad/s = Nms]\ (13)$$

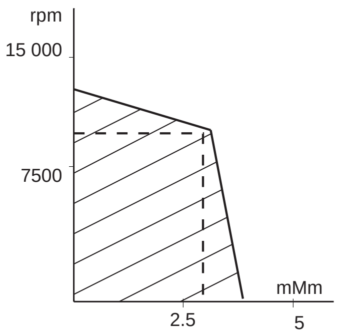
The load torque of 3 mNm requires a current of I = 0.326 A (see formula 1).
The drop in viscous torque due to the lower speed of 10 000 rpm vs 13 400 rpm amounts to:

$DM_f = k_v \cdot D\,w = 0.4 \cdot 10^{-6} \cdot 356 = 0.14$ mNm

This results in a drop in no-load current of 15 mA.
At 10 000 rpm we have:

110 - 15 = 95 mA

When adding them to the load current we arrive at approximately 0.42 A. The rated continuous current of this motor is 0.45 A as defined by the internal overload protection.



Rated working range of the 26BC-A-113 motor and point of actual operation.

The voltage follows formula (2), the voltage drop across the power stage being negligible:

$U = R \cdot I + k \cdot w + u = 2.87 + 9.63 = 12.5$ V

As the drive circuit supply voltage may be from 5V to 18V, the pins 2 and 5 may be hooked together and connected to 12.5 V. If the motor operates but in one direction and there is no speed control, the two wire motor version 26BC-2A offers the simplest solution.

POSITIONING WITH A D.C. MOTOR

A load inertia of $20 \cdot 10^{-7}$ kgm^2 must be moved by an angle of 1 rad in 20 ms. Friction is negligible, ambient temperature is 40°C. With this incremental application we consider a duty cycle of 100% and a triangular speed profile.

Then the motor must rotate 0.5 rad in 10 ms whilst accelerating, then another 0.5 rad in 10 ms whilst braking.

Let's calculate the angular acceleration a:

$$a = \frac{2q}{t^2} \qquad [\text{rad/s}^2]\ (14)$$

$$a = \frac{2 \cdot 0.5}{0.01^2} = 10\,000 \text{ rad/s}^2$$

The torque necessary to accelerate the load is:

$$M_{ch} = J_{ch} \cdot a \qquad [\text{Nm}]\ (15)$$

$$M_{ch} = 20 \cdot 10^{-7} \cdot 10\,000 = 20 \text{ mNm}$$

If the motor inertia equalled the load inertia, torque would be twice that value. We then talk of matched inertias, where the motor does the job with the least power dissipation.

If we consider that case, motor torque becomes:

$$M_m = (J_{ch} + J_m) \cdot a \qquad [\text{Nm}]\ (16)$$

$$M_m = 2 \cdot M_{ch} = 40 \text{ mNm}$$

According to the motor overview the type 28DT12 can deliver 40 mNm permanently. As an example, take the -222E coil with a resistance (at 22°C) of 6.2 W and a torque constant of 32.5 mNm/A. Consider a total thermal resistance of the order of 7.5°C/W. The rotor inertia happens to be just $20 \cdot 10^{-7}$ kgm^2.

From equation (1) we get:

$$I = \frac{M}{k} = \frac{40}{32.5} = 1.23 \text{ A}$$

Equations (9) and (4) give:

$T_r = 143$°C, $R_{Tr} = 9.68$ W

For the triangular profile we then calculate the motor peak speed:

$$w_{max} = a \cdot t \qquad [\text{rad/s}]\ (17)$$

$$w_{max} = 10\,000 \cdot 0.01 = 100 \text{ rad/s}$$

According to equation (3), this gives:

$n_{max} = 955$ rpm

Finally, we apply equation (2):

$$U = R \cdot I + k \cdot w$$
$$= 9.05 \cdot 1.23 + 32.5 \cdot 10^{-3} \cdot 100$$
$$= 15.2 \text{ V}$$

This is the minimum output voltage required by a chopper driver.

A different way of selecting the motor is presented in the technical publication Think escap® 6.

POSITIONING WITH A STEPPER MOTOR

A load inertia of $20 \cdot 10^{-7}$ kg m^2 has to be moved by an angle of 0,5 rad in 20 ms. With a triangular speed profile this requires a torque of 10 mNm up to a peak speed of 50 rad/s as calculated using equations (14) and (15). At that speed the mechanical power for the load alone is 0.5 W. Now we can evaluate the motor size necessary, and we see two possible solutions.

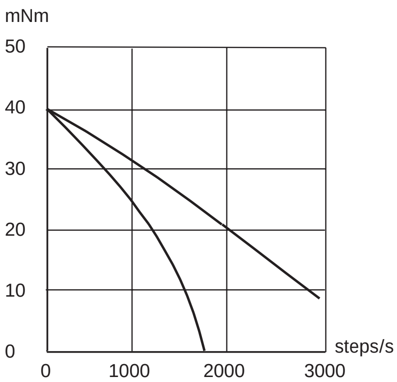
Direct drive

The motor type P430 (100 steps/rev, 60 mNm of holding torque) associated to a simple L/R type driver is quite enough for this application, as peak speed is only 50 rad/s:

$$\frac{50}{2p} \cdot 100 = 796 \text{ steps/s}$$

Let's see whether the move can be done within the motor's pull-in range. Then we would not need to generate ramps for acceleration and deceleration, and the controller would be substantially simplified. In that case we have in fact a rectangular speed profile and the move requires a constant step rate which is obtained by dividing the distance (number of steps which is 8) by the time:

$$\frac{0.5 \cdot 100}{2p \cdot 0.02} = 400 \text{ steps/s}$$



Curves of torque vs step rate for the P430 with ELD-200 drive circuit.

We must make sure the motor can start at that frequency. The curves on page 53 show that, with a load inertia equal to the rotor inertia of 3 gcm^2, the motor can start at about 1700 steps/s. With a load inertia of $20 \cdot 10^{-7}$ kg m^2 this pull-in frequency becomes:

$$f_1 = f_0 \sqrt{\frac{2J_m}{J_m + J_{ch}}} \qquad \text{[Hz] (18)}$$

$$f_1 = 1700 \cdot \sqrt{\frac{6}{23}} = 870 \text{ steps/s}$$

Thanks to the disc magnet technology the P430 motor can do the job quite easily, without needing a ramp, using a very simple controller and an economic driver.

Use of a gearbox

The P310 motor makes 60 steps/rev and has a holding torque of 12 mNm at nominal current. This is too small for moving the load in a direct drive. However, its mechanical power is more than enough. A reduction gearbox can adapt the requirements of the application to the motor capabilities.

Choosing the reduction ratio

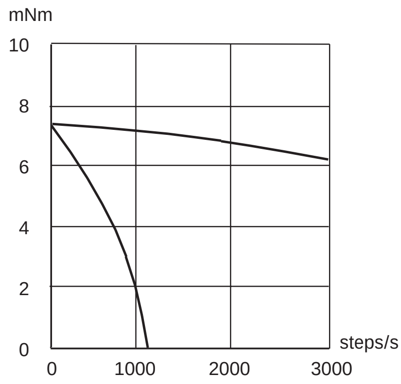
A first choice consists of matching inertias and then make sure that with that ratio the motor speed remains within a reasonable range, where the necessary torque can be delivered. With incremental motion, an inertial match assures the shortest move time, with the motor providing constant torque over the speed range considered. In our example this asks for a ratio i_0 of:

$$i_0 = \sqrt{\frac{J_{ch}}{J_m}} \qquad \text{[-] (19)}$$

$$i_0 = \sqrt{\frac{20}{0.86}} = 4.82$$

From the various gearbox models proposed for the P310 we pick the K24, which offers a smallest ratio of 5:1. Using equations (14), (15) and (19), we find:

- a load inertia reflected to the motor shaft of $1 \cdot 10^{-7}$ kg m^2
- a motor acceleration of 25 000 rad/s^2
- a motor peak speed of 250 rad/s = 2400 rpm = 2400 steps/s
- a necessary motor torque of 5 mNm.



Curves of torque vs step rate for the P310 with ELD-200 drive circuit.

With the ELD-200 drive circuit at 24V the motor P310-158 005, coils in parallel, can do the job with an adequate safety margin. At low step rates the available torque is substantially above the 5 mNm required for the triangular speed profile. By adapting this profile to the motor capabilities the move time can be further reduced.

The smaller P110 motor with R16 gearbox could also do the job but would require a driver of very high performance and carrying a higher price tag.

A detailed description of the disc magnet stepper motor technology is given in the technical publication Think escap® 5.

Designation of ironless rotor D.C. motors

22 N 2R 28 -210E D 16 .1

Motor diameter in mm
Code for motor length
Indication for ball bearings
Commutation system
Winding type
Encoder type
Number of lines of encoder
Motor execution code

Designation of stepper motors

P X 5 3 2 -25 8 012 14 V

Stepper motor
Internal code
Code for diameter
Code for length
Motor version for full/half-step = 2
Motor version for microstep = 0
Number of rotor pole pairs
Number of connections or terminal wires
Resistance per winding (indicated by a letter for some motors)
Motor execution code
Particular option

Designation of BLDC motors, slotless iron structure

26 BC 6 A 107 .101

Motor diameter
Technology
Number of terminal wires
Commutation system
Winding type
Execution code

Designation of BLDC motors, slotted iron structure

B 0508 -050A -R 0 G 05 F

Motor type (B = Brushless motor)
Diameter & length in inches (0508 = 0.5" D x 0.8" L)
Nominal voltage (050A/050B/150A/150B)
Motor shaft options (R = round, F = flat, D = double, O = gearhead)
Mounting options (0 = threads, 1 = servo groove, 2/3/4 = screw diamond/triangle/square)
Configuration (M = just motor, G = gearhead)
Gear ratio (05 = 5:1)
Gearhead shaft options (R = round, F = flat, D = double)

Example of gearboxes designation

R 22 0- 190

Gearbox type
Gearbox diameter in mm
Gearbox execution code
Reduction ratio

Example of gearmotors designation

M 707L61-207 10.7 .0

Gearmotor
Motor type and definition
Reduction ratio
Gearmotor execution code